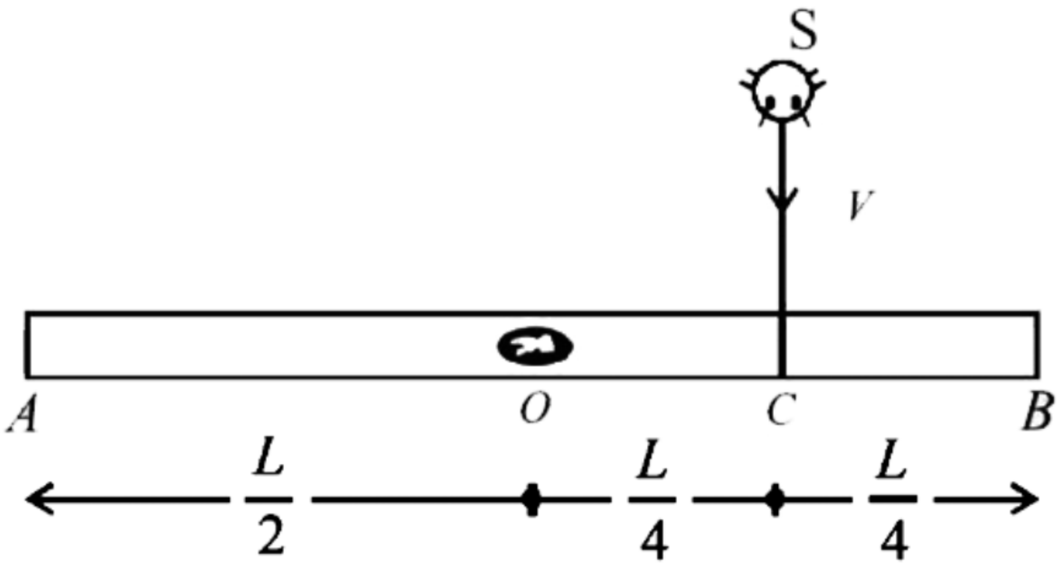
IIT JEE 1992 Question

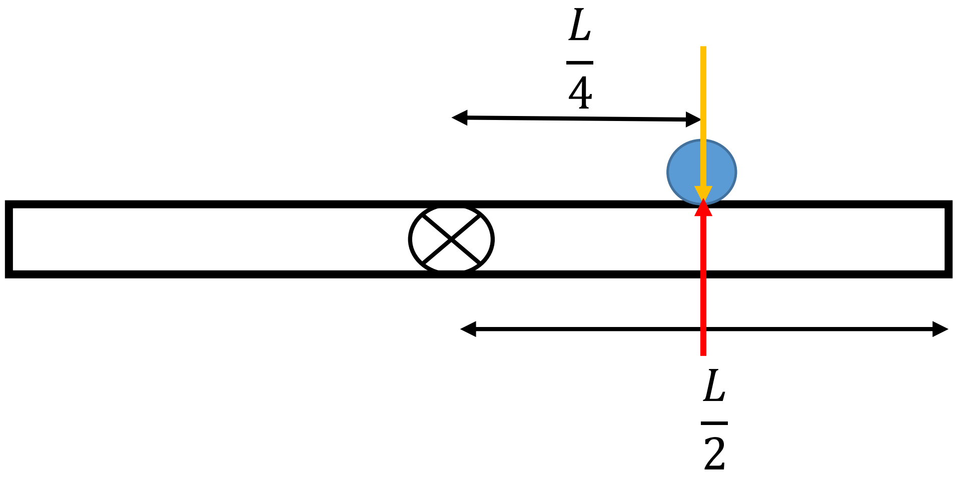
- A homogeneous rod AB of length L = 1.8 m and mass M is pivoted at the center O in such a way that it can rotate it can rotate freely position. An insect S of the same mass M falls vertically with speed V on the point C, midway between the points O and B. Immediately after falling, the insect moves towards the end B such that the rod rotates with a constant angular velocity omega.
  (a) Determine the angular velocity ω in terms of V and L.
  (b) If the insect reaches the end B when the rod has turned through an angle of 90∘, determine V.



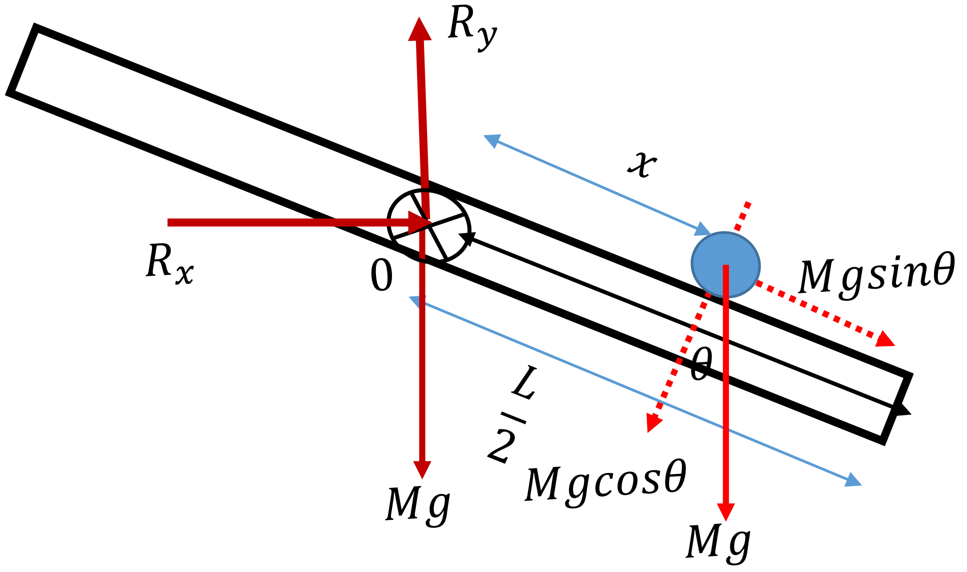
- Angular momentum before collision

$= \frac{MVL}{4}$

- Moment of inertia of rod = $\frac{ML^2}{12}$
- Moment of inertia of insect = $M(\frac{L}{4})^2$
- Total Moment of inertia about rod center = $\frac{ML^2}{12} + \frac{ML^2}{16}$
- Applying conservation of angular momentum
- $\left(\frac{ML^2}{12} + \frac{ML^2}{16}\right)\omega = \frac{MVL}{4}$
- $\omega = \frac{12V}{7L}$



- Total moment of inertia about rod center. I = $\frac{ML^2}{12} + Mx^2$
- Angular momentum of system
- L=I ω
- Taking Torque about 0 = $Mgcos\theta * x$
- Torque = $\frac{dL}{dt} = \frac{d}{dt}(\text{I}\ \omega)$
- $\omega \frac{d}{dt}\left(\frac{ML^2}{12} + Mx^2\right) = \omega \frac{d}{dt}(0 + Mx^2)$
- $\omega \frac{d}{dt}(Mx^2) = \omega M \frac{d}{dt}(x^2) = 2x\omega M \frac{dx}{dt}$
- $2x\omega M \frac{dx}{dt} = Mgcos\theta * x$



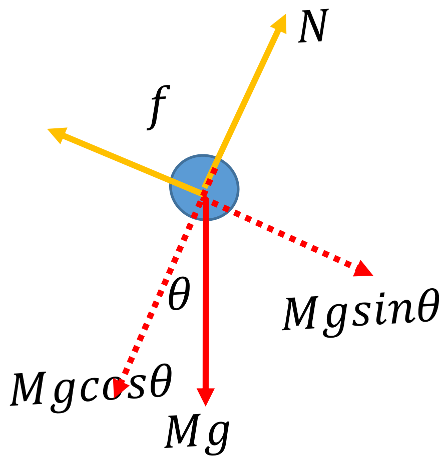
- $2x\omega M\frac{dx}{dt} = Mgcos\theta * x$
- $2x\omega M\frac{dx}{d\theta}\frac{d\theta}{dt} = Mgcos\theta * x$

- $2x\omega M\frac{dx}{d\theta}\omega = Mgcos\theta * x$
- $2\omega^2 dx = gcos\theta * d\theta$
- $\int_{L/4}^{L/2} 2\omega^2 dx = \int_0^{\pi/2} gcos\theta \, d\theta$
- $\frac{2\omega^2 L}{4} = g$
- $\omega^2 = \frac{2g}{L}$
- $\omega = \sqrt{\frac{2g}{L}}$

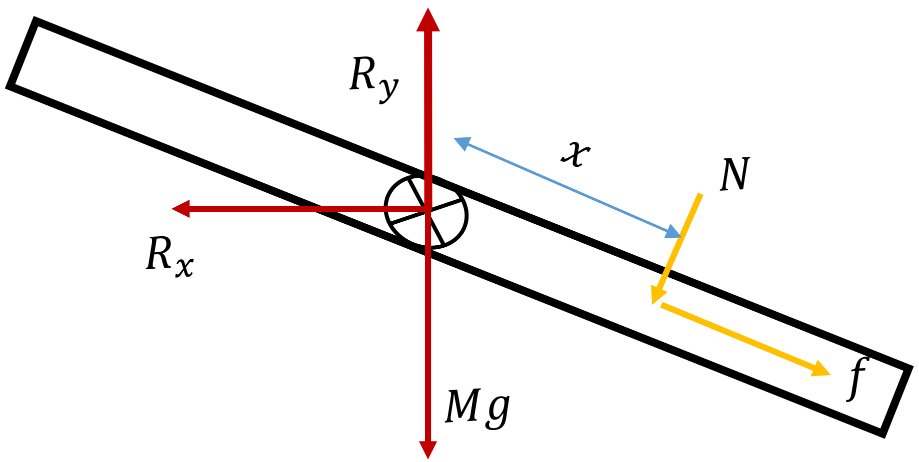
- $\omega = \frac{12V}{7L}$

- $V = \frac{7}{12}\sqrt{2 * 10 * 1.8}$
- V =3.5m/s

- $a_r = \ddot{r} - \omega^2 r$
- $a_\theta = \ddot{\theta} + 2\omega\dot{r}$
- $F_r = Ma_r$
- $F_\theta = Ma_\theta$
- $f - Mgsin\theta = \ddot{r} - \omega^2 r$
- $Mgcos\theta - N = \ddot{\theta} + 2\omega\dot{r}$



- Moment of inertia about rod center. $I_0 = \frac{ML^2}{12}$
- Angular momentum of system
- L=$I_0\ \omega$
- Torque = $\frac{dL}{dt} = \frac{d}{dt}(I_0\omega)$
- $N * x = \frac{d}{dt}(I_0\omega)$
- $N * x = 0$
- N=0



- $Mgcos\theta - N = M(\ddot{\theta} + 2\omega\dot{r})$
- $N = 0$ and $\ddot{\theta} = 0$
- $gcos\theta = 2\omega\dot{r}$
- $gcos\theta = 2\omega\frac{dx}{dt}$
- $gcos\theta = 2\omega\frac{dx}{d\theta}\frac{d\theta}{dt}$
- $gcos\theta = 2\omega\frac{dx}{d\theta}\omega$
- $2\omega^2 dx$= $gcos\theta * d\theta$
- $\int_{L/4}^{L/2} 2\omega^2 dx = \int_0^{\pi/2} gcos\theta\, d\theta$
- $\frac{2\omega^2 L}{4} = g$
- $\omega^2 = \frac{2g}{L}$
- $\omega = \sqrt{\frac{2g}{L}}$
- $\omega = \frac{12V}{7L}$
- $V = \frac{7}{12}\sqrt{2 * 10 * 1.8}$
- V =3.5m/s